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Rigid Quantum Secret Sharing Based on Cavity QED Through an Amplitude-Damping Noisy Channel

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Abstract: A proposed model of quantum transmission of a secret message through an amplitude-damping noisy environment via Fock cavity field is achieved. The process is based on the contribution of a cavity QED, to have a rigidity against the eavesdropping attacks. Indeed, the interaction time, the strength coupling and the Fock field number also control the successful probability of the quantum secret sharing, which makes the interception messages more difficult during our proposed model.

Keywords: noisy environment, Fock cavity field, successful probability, quantum secret sharing.

1 Introduction

Quantum entanglement has been attracted much attention because of its power in quantum information processing as a special resource. In this context, many quantum tasks can be accomplished by quantum entangled states [1], such as quantum key distribution [2, 3], quantum teleportation [4], quantum secure direct communication [5, 6], quantum secret sharing [7-10], remote state preparation[11], and quantum state sharing (QSTS) [12] where a secret state is unknown to the sender and shared among a set of agents. We add that in the QSTS process only qualified agents groups can contribute to recover the state, which has a strong relationships with quantum teleportation and RSP.

In the real world, during the entanglement applications, the entangled resources have been generated and transmitted by interaction with the outside environment. These interactions are considered as noises. In this context, many works have achieved the quantum communication through a noisy environment [13-16]. In the presence of noises, during the QSTS schemes, the information is affected and it will be a lost of information. Therefore, the fidelity will depend so on the noise rate parameter. Then, when this parameter becomes smaller, the fidelity keeps its higher value whatever the amplitude of the unknown state, but in the small noise rates case, the fidelity changes its value as a function of the amplitude of the unknown state [10]

The present paper enhances a new QSS scheme [10], which is presented in an amplitude-damping noisy quantum channel, by exploiting the electrodynamics interaction based on cavity QED. This enhancement is analyzed by studying its robustness after adding the effect of a Fock cavity field in the secret recovery phase. The proposed schemes exploit the cavity QED to improve the previous scheme [10] in order to avoid the possibility of any eavesdropping attack by adding an additional controller parameters in the QSS scheme as the interaction time, the strength coupling and the Fock field number. However, we keep a higher fidelity value by adjusting these parameters only in some periodic values. This period will be unknown for the eavesdropper and changes its value according to the amplitude-damping noise, which makes any attack more difficult to be achieved successively. The current work achieves the same study purpose in the case of amplitude-damping noisy channels as it is done in [17] in the case of phase-damping noisy channels study.

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2 Without cavity QED use, how was the QSS scheme in an amplitude-damping noisy environment?

2.1 The amplitude damping noise

The loss of energy from a quantum system causes an energy dissipation rate according to the kind of the noise action. In this context, the amplitude-damping noise is one of the most important decoherence noises, its action is described by a set of Kraus operators as [18]

$$E_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\lambda} \end{pmatrix}, \qquad (1)$$

$$E_1 = \begin{pmatrix} 0 & \sqrt{\lambda} \\ 0 & 0 \end{pmatrix}, \qquad (2)$$

Where, $0 \le \lambda \le 1$ is the probability that a quantum state will be affected after passing through a noisy channel, and λ is called also the decoherence rate of the amplitude-damping noisy environment.

Consider that Alice wants to distribute a secret state between two agents Bob and Charlie in such way that only if two agents work together they can recover the quantum state. The arbitrary single-qubit state to be shared has supposed in this form

$$|\psi\rangle = a_0 e^{i\theta_0} |0\rangle + a_1 e^{i\theta_1} |1\rangle \qquad (3)$$

Knowing that a_0, a_1 are a real numbers known by Alice and

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satisfying $a_0^2 + a_1^2 = 1$, also θ_0, θ_1 are supposed known by Alice satisfying this condition $\theta_0, \theta_1 \in [0, 2\pi]$.

The scheme of sharing an arbitrary unknown qubit state can be applied after two steps; the secret splitting phase and the secret recovery phase.

2.2 The secret splitting phase in the scheme

In this step, Alice prepares a GHZ state $|\phi\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle_{ABC} + |111\rangle_{ABC}\right)$

As shared quantum resource. The density matrix of this state is expressed as

$$\rho_{ABC} = |\phi\rangle\langle\phi| \quad (4)$$

Alice keeps the qubit A and sends qubits B and C to Bob and Charlie through two identical amplitude-damping noisy channels, (its effect is described by Eq.(1),(2)). This process will change the pure channel state into a mixed one. After qubits transmissions through the amplitude-damping noisy channels, the quantum resource which is used for state sharing becomes

$$\begin{aligned}\varepsilon(\rho) &= \sum_{i,j=0,1} \mathrm{E}_i^B \mathrm{E}_j^C \rho \left(\mathrm{E}_i^B\right)^\dagger \rho \left(\mathrm{E}_j^C\right)^\dagger \\ &= \frac{1}{2}\Big[\left(|000\rangle + (1-\lambda)|111\rangle\right)\left(\langle 000| + (1-\lambda)\langle 111|\right)\Big] \\ &\quad + (1-\lambda)\lambda|110\rangle\langle 110| + (1-\lambda)\lambda|101\rangle\langle 101| + \lambda^2|100\rangle\langle 100| \quad (5)\end{aligned}$$

Knowing that superscripts B and C present noise operators action on qubits B and C, respectively, and, † means the conjugate transpose.

Secondly, Alice prepares an ancilla state R in $|0\rangle$ and she performs an unitary operation U on particles AR. Then, the quantum system will be

$$\rho_1 = U_{AR} \otimes \mathrm{I}_{BC}\left\{\varepsilon(\rho)_{ABC} \otimes |0\rangle\langle 0|_R\right\} U_{AR}^\dagger \otimes \mathrm{I}_{BC} \quad (6)$$

Where each subscript indicates a qubit,

Thirdly, Alice measures qubit R in the computational basis. If she gets the result $|0\rangle$ by using as measurement operator $M_0 = |0\rangle\langle 0|$ so the quantum system becomes as follows

$$\begin{aligned}\rho_2 &= tr_R\left(\frac{M_0 \rho_1 M_0^\dagger}{tr\left(M_0 M_0^\dagger \rho_1\right)}\right) \\ &= \left(a_0|000\rangle + a_1(1-\lambda)|111\rangle\right)\left(a_0\langle 000| + a_1(1-\lambda)\langle 111|\right) \\ &\quad + a_1^2(1-\lambda)\lambda|110\rangle\langle 110| + a_1^2(1-\lambda)\lambda|101\rangle\langle 101| + a_1^2\lambda^2|100\rangle\langle 100| \quad (7)\end{aligned}$$

Else, if Alice gets the result $|1\rangle$, she needs to use the recursive way.

Next, Alice performs a projective measurement on the qubit A under the basis $\{|\Lambda_k\rangle; k \in \{0,1\}\}$ Consider that the measurement result is $|\Lambda_k\rangle$ by using as measurement operator $M_{\Lambda_k} = |\Lambda_k\rangle\langle\Lambda_k|$ with $k \in \{0,1\}$, the quantum system will be rewritten as follows

$$\begin{aligned}\rho_3 &= \frac{M_{\Lambda_k}\rho_2 M_{\Lambda_k}^\dagger}{tr\left(M_{\Lambda_k} M_{\Lambda_k}^\dagger \rho_2\right)} \\ &= \frac{1}{2}[(a_0|000\rangle + (-1)^k a_1(1-\lambda)e^{-i\theta_0}e^{i\theta_1}|011\rangle + (-1)^k a_0 e^{i\theta_0}e^{-i\theta_1}|100\rangle \\ &+ a_1(1-\lambda)|111\rangle) \times (a_0\langle 000| + (-1)^k a_1(1-\lambda)e^{i\theta_0}e^{-i\theta_1}\langle 011| \\ &+ (-1)^k a_0 e^{-i\theta_0}e^{i\theta_1}\langle 100| + a_1(1-\lambda)\langle 111|) \\ &+ a_1^2(1-\lambda)\lambda((-1)^k e^{-i\theta_0}e^{i\theta_1}|010\rangle + |110\rangle)((-1)^k e^{i\theta_0}e^{-i\theta_1}\langle 010| \\ &+ \langle 110|) + a_1^2(1-\lambda)\lambda((-1)^k e^{-i\theta_0}e^{i\theta_1}|001\rangle + |101\rangle)((-1)^k e^{i\theta_0}e^{-i\theta_1}\langle 001| \\ &+ \langle 101|) + a_1^2\lambda^2((-1)^k e^{-i\theta_0}e^{i\theta_1}|000\rangle \\ &+ |100\rangle)((-1)^k e^{i\theta_0}e^{-i\theta_1}\langle 000| + \langle 100|)] \quad (8)\end{aligned}$$

Then, the quantum system of Bob and Charlie becomes

$$\begin{aligned}\rho_4 &= tr_A(\rho_3) \\ &= a_0^2|00\rangle\langle 00| + ((-1)^k a_0 e^{i\theta_0}e^{-i\theta_1}(1-\lambda)|00\rangle\langle 10| \\ &+ ((-1)^k a_0 e^{-i\theta_0}e^{i\theta_1}(1-\lambda)|11\rangle\langle 00| + a_1^2(1-\lambda)^2|11\rangle\langle 11| \\ &+ a_1^2(1-\lambda)\lambda|01\rangle\langle 01| + a_1^2(1-\lambda)\lambda|10\rangle\langle 10| + a_1^2\lambda^2|00\rangle\langle 00| \quad (9)\end{aligned}$$

2.3 The secret recovery phase in the scheme

Up to now, Bob and Charlies quantum system is ρ_4. Consider that Bob agrees to cooperate with Charlie in order to recover the secret state. Here, Bob performs a single-qubit measurement under the basis $\{|0\rangle, |1\rangle\}$.

Bob using the measurement operator $M_{B_{k_1}} = |k_1\rangle\langle k_1|$ with $k_1 = \{0,1\}$ applies its measurement. So, the state shared by Charlie will takes this form

$$\begin{aligned}\rho_C &= tr_B\left(\frac{M_{B_{k_1}}\rho_4 M_{B_{k_1}}^\dagger}{tr\left(M_{B_{k_1}} M_{B_{k_1}}^\dagger \rho_4\right)}\right) \\ &= (a_0^2 + a_1^2\lambda)|0\rangle\langle 0| + a_1^2(1-\lambda)|1\rangle\langle 1| + (-1)^{k+k_1} a_0 e^{i\theta_0}e^{-i\theta_1} a_1(1-\lambda)|0\rangle\langle 1| \\ &+ (-1)^{k+k_1} a_0 e^{-i\theta_0}e^{i\theta_1} a_1(1-\lambda)|1\rangle\langle 0| \quad (10)\end{aligned}$$

Finally, Bob sends his measurement result k_1 to Charlie.

Then, Charlie recover the secret state by performing $\sigma_z^{k+k_1}$ on qubit C. The recovery state so has the form

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$$\rho_{out} = \sigma_z^{k+k_1} \rho_C \left(\sigma_z^{k+k_1}\right)^{\dagger}$$
$$= (a_0^2 + a_1^2\lambda)|0\rangle\langle 0| + a_1^2(1-\lambda)|1\rangle\langle 1| + a_0 e^{i\theta_0} e^{-i\theta_1} a_1(1-\lambda)|0\rangle\langle 1|$$
$$+ a_0 e^{-i\theta_0} e^{i\theta_1} a_1(1-\lambda)|1\rangle\langle 0| \qquad (11)$$

2.4 The scheme fidelity

The noisy environment will affect of course the initial state $|\psi\rangle$ to be as another mixed one described by ρ_{out}. For this reason, the fidelity is calculated to know the difference between both states as follows

$$F_{AD} = \langle\psi|\rho_{out}|\psi\rangle = a_0^4 + a_0^2 a_1^2 (2-\lambda) + a_1^4(1-\lambda)$$

It appears clearly in the figure 1 that in the case of an amplitude-damping noisy environment, during the QSS the fidelity depends strongly on the amplitudes of the prepared state $|\psi\rangle$ and the noise rate λ.

2.4 Discussion

The fidelity value remains perfect and maximal (equal to 1), when the amplitude of the initial state $|\psi\rangle$ is found maximal (a_0=1 or a_1=1: no superposition in the initial atomic state is seen) whatever the noise rate λ. Also, in case when the noise rate λ is found maximal whatever the initial state $|\psi\rangle$.

It means that the QSS scheme through an amplitude-damping noisy channel without cavity QED use has a perfect successful probability only when the initial state $|\psi\rangle$ is non-superposed.

In addition, an ideal quantum channel can gives a perfect successful probability of the QSS scheme without carrying about the initial state $|\psi\rangle$.

The mean problem of such schemes appears in its weak security, which comes from the standard behaviour of fidelity (figure 1). That's gives to the eavesdropper a prior idea on the way in which he will act to get the initial message according to the noise rate. In this context, the next section overtakes this problem based on Fock cavity field use.

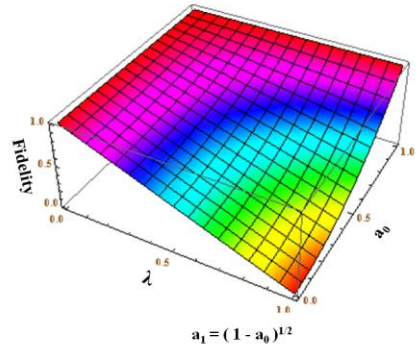

Figure 1: The QSS fidelity in case of an amplitude-damping noisy environment scheme without cavity QED use.

3 The QSS scheme in an amplitude-damping noisy environment using cavity QED

Many works have appended the cavity QED use to exploit the electrodynamics interaction in order to achieve and to enhance the quantum communication protocols [19]. In the following, we enhance the previous QSS scheme in the same amplitude-damping noisy environment using a Fock cavity field.

3.1 The proposed scheme

Here, we keep all the process steps in the previous section 1 until the time when Charlie performed the $\sigma_z^{k+k_1}$ operator on the Bob's measurement to recover the state in Eq. (11). But, we consider here that in the final step, Charlie sends its mixed state ρ_{out} to a cavity supposed experimentally of a non-leaky type [20-23].

The coupled atom-field system is described by the Jaynes-Cummings Hamiltonian. Thus, the interaction Hamiltonian in a rotating frame at the cavity mode frequency and in the rotating wave approximation, at exact resonance, can be expressed as

$$\mathrm{H} = \hbar g\left(\sigma_+ a + a^{\dagger}\sigma_-\right) \qquad (12)$$

Where $a\left(a^{\dagger}\right)$ indicate the annihilation (creation) operators of the single-mode cavity field and $\sigma_+\left(\sigma_-\right)$ denote the raising (lowing) operators of the atom, and (g) is the atom-field coupling constant. By assuming that the cavity field is initially in the n-photons Fock state $|n\rangle$ and knowing that the atom having the mixed state ρ_{out} pass through the cavity. The total system will be initially found in the tensor product state $\rho_{out}(0) = \rho_{out} \otimes |n\rangle\langle n|$

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$$\rho_{out} = \begin{pmatrix} \rho_{out_{11}} & \rho_{out_{12}} \\ \rho_{out_{21}} & \rho_{out_{22}} \end{pmatrix} \quad (13)$$

As defined in Eq. (11)

$$\rho_{out_{11}} = (a_0^2 + a_1^2\lambda), \quad (14)$$

$$\rho_{out_{12}} = a_0 e^{i\theta_0} e^{-i\theta_1} a_1 (1-\lambda) \quad (15)$$

$$\rho_{out_{21}} = a_0 e^{-i\theta_0} e^{i\theta_1} a_1 (1-\lambda) \quad (16)$$

$$\rho_{out_{22}} = a_1^2 (1-\lambda) \quad (17)$$

The dynamics of the atomic-field state after interaction with the cavity will be gotten by using the master equation

$$\rho_{out}(t) = U(t)\rho_{out}(0)U^{\dagger}(t), \quad (18)$$

Knowing that $U(t) = e^{-i\frac{Ht}{\hbar}}$ is the evolution operator. After making the partial trace on the Fock field subspace, we get

$$\begin{aligned} \rho_{out2} = tr_F(\rho_{out}(t)) = &|0\rangle\langle 0|\left(\cos^2(gt\sqrt{n+1}\times\rho_{out_{11}}\right) \\ &+|1\rangle\langle 1|\left(\cos^2(gt\sqrt{n+1}\times\rho_{out_{22}}\right) \quad (19) \\ &+|0\rangle\langle 1|\left(n\times\rho_{out_{21}}\times\frac{\sin^2(gt\sqrt{n+1}}{n+1}+\cos^2(gt\sqrt{n+1})\times\rho_{out_{12}}\right) \end{aligned}$$

Thus, in the end of this scheme, Charlie will get the state (20) instead of the state (11) comparing with the first scheme. Let's discuss then the fidelity behaviour she will get here to compare it with the previous scheme fidelity.

3.1 The scheme fidelity

In this scheme, the initial state $|\psi\rangle$ will be affected by not only the noisy environment but also the effect of the Fock cavity field such as the interaction time and the coupling constant, to have as a consequence ρ_{out2} then, the fidelity is calculated to know the difference between both states as follows

$$\begin{aligned} F_{ADC} &= \langle\psi|\rho_{out2}|\psi\rangle \\ &= \frac{1}{1+n}e^{-i(2\theta_0\theta_1)}(-\left(1-a_0^2\right)e^{i(2\theta_0\theta_1)}(1+n)(-1+\lambda)\cos^2(gt\sqrt{n+1}) \quad (20) \\ &+a_0^2 e^{i\theta_1}\left(e^{2i\theta_0}(1+n)\cos^2(gt\sqrt{n+1})+\left(-1+a_0^2\right)e^{2i\theta_1}(-1+\lambda)\sin^2(gt\sqrt{n+1})\right)) \end{aligned}$$

Knowing that we replace in the expression above a_1 by $\sqrt{1-a_0^2}$

In the following, in figures (2, 3), we give the plots of the current QSS scheme fidelity corresponding to two different values of the Fock field number.

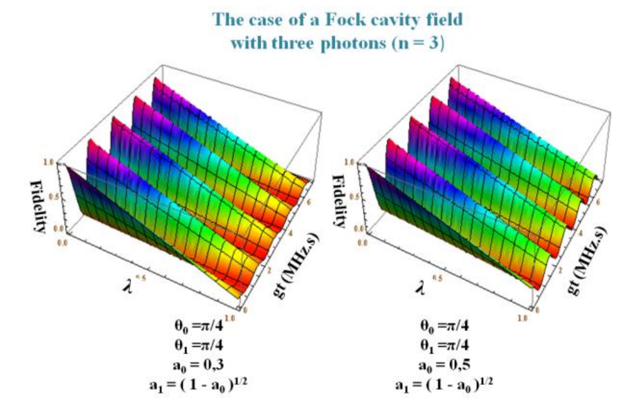

Figure 2: The QSS scheme fidelity in an amplitude-damping noisy environment with cavity QED use, noting that three photons were initially in the Fock field.

Furthermore, in two dimensions, in order to study the two cases when the channel is ideal (λ=0) and when the channel is noisy (λ=1), we have plot the fidelity as a function of the product between the coupling constant and the interaction time (which is called the scaled time

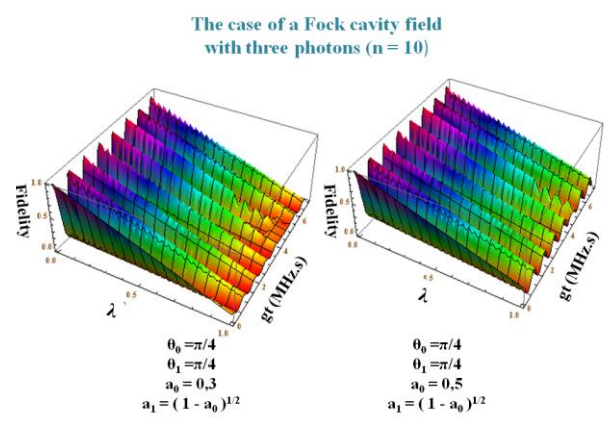

Figure 3: The QSS scheme fidelity in an amplitude-damping noisy environment with cavity QED use, noting that ten photons were initially in the Fock field.

(Which is called the scaled time T=gt) The figures (4, 5) so show the fidelity behaviours for two diferent values of the Fock cavity field number.

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3.3 Discution

Starting from the figure 6, when the initial state $|\psi\rangle$ is non-superposed ($a_0 = 1$ or $a_1 = 1$), whatever the value of the noise rate λ, the fidelity of the QSS scheme using cavity QED reaches its maximal value in a periodic values of the scaled time (T=gt).

This periodicity becomes faster (more peaks are seen in the fidelity behaviour), when Charlie increases the Fock field number n inside the cavity.

In addition, whatever the used channel kind (noisy or ideal), using a superposed initial state $|\psi\rangle$, for different values of the $|\psi\rangle$ amplitude, the fidelity of the QSS scheme reaches its maximum value in some periodic values of the scaled time

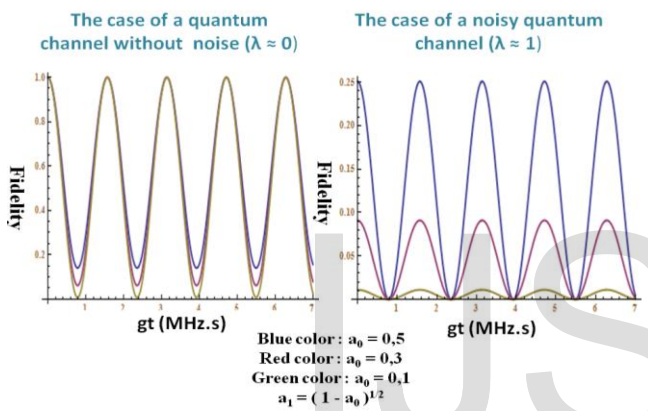

Figure 4: The QSS fidelity in case of cavity QED use where $\theta_0 = \theta_1 = \frac{\pi}{4}$ three photons are initially in the Fock field.

(T=gt). Moreover, Charlie can enhance this maximum value of the fidelity by using a smaller noisy rate. We add that she can control also the frequency of successful recovering of the initial state by adjusting a suitable Fock cavity field n, the fidelity period becomes shorter in a higher used Fock cavity field n.

3 Conclusion

By inserting a cavity QED, especially a Fock cavity field, our QSS scheme becomes rigid against any eavesdropping attacks. Indeed, the eavesdropper who wants to intercept the message must have sequence values of the periodic scaled time wherein the fidelity of the message transmission is maximal. This condition is difficult to be satisfied, because the third users (Charlie) control this sequence values by her choice of the used Fock cavity field number n.

Then, the eavesdropper must have in addition the period of the scaled time to

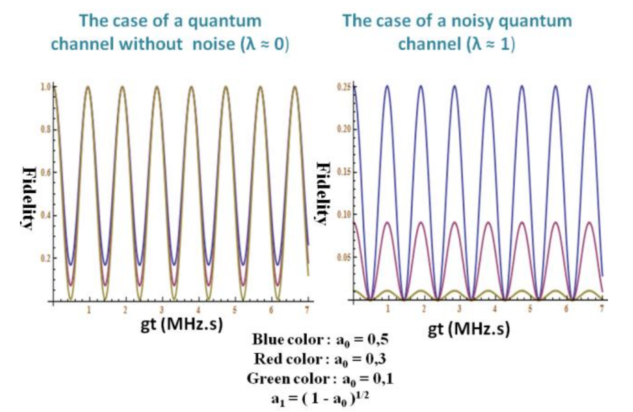

Figure 5: The QSS fidelity in case of cavity QED use where $\theta_0 = \theta_1 = \frac{\pi}{4}$ ten photons are initially in the Fock field.

know in which time he has to intercept the message (he has to apply his attack in the peaks of the figures bellow 2-6). That's the mean difficulty for the eavesdropper. Especially, he will find also another unknown parameters more than the cavity parameters, which complicates more the eavesdropping process.

The discussion of the scheme indicates that it can be realized by the current technology, which gives more efficiency to our work.

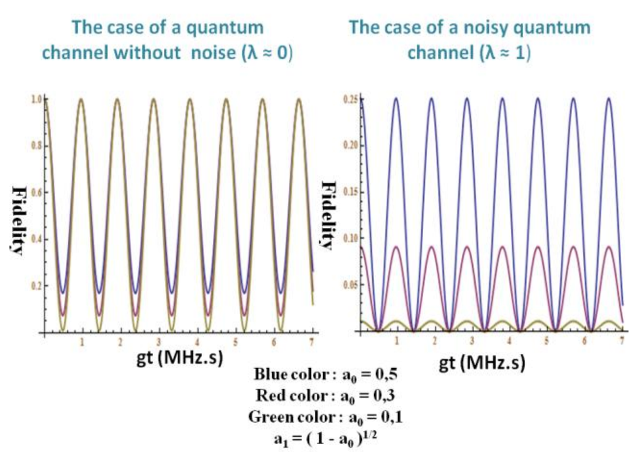

Figure 6: The QSS fidelity in case of a scheme including an amplitude-damping noisy environment with cavity QED use, the initial state is non-superposed ($a_0 = 1$ and $a_1 = 0$).

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